ticularly useful as a resonator for which a very high Q value and excellent transmission are required. The Q values of parallel plate resonators can be described by the well-known expression

$$Q = m \frac{\pi}{2} \frac{1 + \Gamma^2}{1 - \Gamma^2}$$

where m stands for the order of interference, given by the separation of the reflector plates, measured in units of half wavelengths. The parameter $\Gamma^2 = 1\text{-}a\text{-}t\text{-}d\text{-}r$ is determined by the various possible power losses as defined below.

a = power absorption coefficient of one reflector plate
t = power transmission coefficient of one reflector plate
d = power loss per single path due to diffraction
r = power loss per single path due to imperfections in flatness of the reflector plate which causes "off axis" reflections.

The increase of Q values by working at a relatively high order of interference, is limited in the mm-wave region for practical reasons, since for $m > 100$ the diameter of the reflector plates becomes inconveniently large so that diffraction losses can be neglected. By choosing the transmission t equal to the absorption of silver-plated perforated metal plates, maximum Q/m values

$$\left(Q/m = \frac{\pi}{2} \frac{1 + \Gamma^2}{1 - \Gamma^2}\right)$$

in the order of 2000 can be predicted.

Provided diffraction losses can be neglected by working with sufficiently large plate diameters, deviations from this Q/m value can be attributed to imperfections of the plate flatness, specified by the parameter r. With increasing plate separation, the flatness requirements become more severe.

It can be estimated that in the mm-wave region Fabry-Perot reflector plates should be flat within 1μ to provide Q/m values of approximately 2000 at a plate separation of 50 half wavelengths. For this plate separation, the plate diameter should be in the order of 200 mm[6] so that diffraction losses can be neglected, and thus it will be rather difficult to satisfy the flatness requirements.

This paper describes the results of various fabrication techniques used to make perforated metal plates[1] which meet the flatness requirement of mm-wave Fabry-Perot reflector plates and which are comparatively insensitive to mechanical and thermal influences. All measurements were performed at 70 Gc. The diameter of each plate was chosen to be 220 mm and the distance between the coupling holes and their diameter was 2.5 and 1.25 mm, respectively. With a plate thickness of 1 mm, the transmission of a single plate was measured to be 32 db. Except as otherwise stated, all Q/m values were measured for $m = 50$. With perforated aluminum plates glued on supporting rings, Q/m values could be achieved between 300 and 400.

[6] A. G. Fox and T. Li, "Resonant modes in a maser interferometer," *Bell Sys. Tech. J.*, vol. 40, pp. 435–488; March, 1961.

To show the influence of an insufficient flatness of the resonator plates, an attempt was made to stretch the plates in a manner similar to stretching the skin of a drum. One simple way to apply a homogeneous stretching is the use of thermal expansion forces. By heating the supporting rings electrically, Q/m values could be increased to 800.

Instead of heating, we have also glued aluminum plates at temperatures of 50°C on supporting rings of stainless steel. Since steel has a smaller thermal expansion coefficient than aluminum, the plate will again be stretched if the system has cooled down to room temperature. Again Q/m values in the order of 800 have been measured. These values are still smaller than those predicted theoretically. An investigation of the reflector plate flatness by means of an optical interferometer proved that the surface imperfections are still in the order of 10μ. Lapping and polishing methods were tried to correct the imperfections, but were unsuccessful because of an insufficient rigidity of the plates. To increase the rigidity of the plates, only the coupling area of a one-inch thick aluminum plate was machined down to a thickness of 1 mm. The whole plate consisted of the same material to avoid mechanical strains due to temperature changes.

The diameter of the coupling area was 125 mm and consisted of 2500 equally spaced coupling holes. After the plates had been electrolytically silver plated, each was lapped and polished while the flatness was consistently controlled with an optical interferometer. Thus, it was possible to achieve a flatness of 1μ over the entire surface area of the reflector plate.

Measured Q/m values of 1800 for these plates are close to those predicted theoretically. The diagram of Fig. 1 depicts the measured Q-values and the insertion loss for a Fabry-Perot resonator with the above-described reflector plates as a function of the plate separation and thus the order of interference. The highest Q value of 160,000 was measured at the 110th order of interference. For larger reflector plate distances, energy losses due to diffraction effects will be so significant that the Q value will decrease. The insertion loss increases with an increased plate separation; at Q values of 100,000 the insertion loss was measured to be 13 db.

It should be mentioned, however, that because of the unfavorable thickness to diameter ratio of the coupling area, the lapping of the described reflector plates is a rather lengthy and cumbersome task. In this respect, metal evaporated quartz plates seem to have very promising features, since they can be lapped with much less effort to the required flatness limits.

Therefore, in our Fabry-Perot resonator, copper evaporated quartz plates having a diameter of 220 mm and flatness imperfections less than $1/10\mu$ were also checked. The thickness of the copper film was larger than 3 skin depths; the distance between the center of the photo-etched coupling holes was the same as that of the above described metal plates; the diameter of the holes, however, had to be decreased to 0.5 mm in order to achieve again a transmission of 32 db. Despite the improved flatness of the

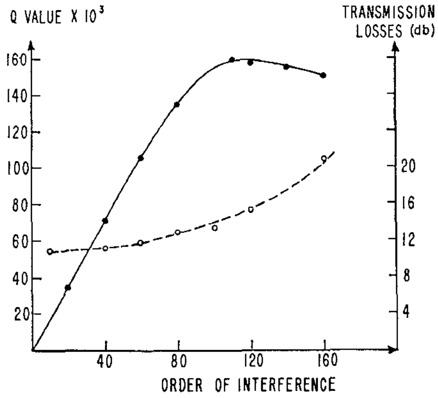

Fig. 1—Q values and transmission losses of a Fabry-Perot resonator vs the order of interference.

quartz reflector plates, all Q/m values have been measured to be slightly smaller than those of solid metal plates. These negative results can possibly be attributed to increased absorption losses caused by a slight deviation of the quartz plate thickness from the exact resonance condition as described by Zimmerer.[5]

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A Differential Microwave Phase Shifter

It is well known[1,2] that a standard phase shifter can be constructed by combining a short-circuited section of uniform waveguide and a tuned, single directional coupler type of reflectometer, as shown in Fig. 1. In operation, the phase of the side arm output tracks the position of the short-circuiting plunger, once the tuning adjustments have been correctly made. The phase change ϕ corresponding to a displacement l of the short circuit is

$$\phi = 2\beta l \tag{1}$$

where $\beta = 2\pi/\lambda_G$ and λ_G is the "guide wavelength."

The purpose of this note is to suggest a method of obtaining a differential phase shifter by combining two phase shifters of the above type with ganged short circuits, as shown in Fig. 2. The phase shifters are arranged so that the phase shift ψ of the output is the difference of the phase shifts produced by the two units, or $\psi = \phi_2 - \phi_1$. If the uniform waveguide sections in which the short circuits slide have identical cross sections, then $\phi_1 = \phi_2$ and the phase shift, $\psi = 0$.

Manuscript received November 6, 1963.

[1] M. Magid, "Precision microwave phase shift measurements," IRE Trans. on Instrumentation, vol. I-7, pp. 321–331; December, 1958.

[2] G. E. Schafer and R. W. Beatty, "Error analysis of a standard microwave phase shifter," *J. Res. NBS*, vol. 64C, pp. 261–265; October–December, 1960.

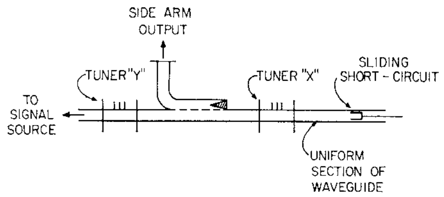

Fig. 1—A type of standard phase shifter.

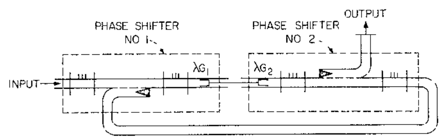

Fig. 2—Differential phase shifter.

But if one waveguide section has a different cutoff wavelength λ_c from the other, then the phase shift ψ is not equal to zero; instead,

$$\psi = 4\pi l \left(\frac{1}{\lambda_{G_2}} - \frac{1}{\lambda_{G_1}}\right) \tag{2}$$

where the guide wavelength λ_G is related to the cutoff wavelength λ_c by

$$\frac{1}{\lambda_G^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}.$$

The cutoff wavelengths of the two waveguides can be chosen to produce any phase shift ψ between zero and the limiting case when one waveguide is operating below cutoff and the phase shift ψ is that of a single phase shifter alone.

Such a differential phase shifter has a number of potential applications such as the following. As the above standard phase shifter is extended to higher frequencies, say above 30 Gc, it takes a smaller displacement to produce the same phase shift; hence, errors in determining this displacement produce correspondingly larger errors in the phase shift. This situation can be avoided by using a differential phase shifter as described above, with the waveguide cutoff frequencies chosen so that the phase of the output varies more slowly than it would if it were tracking the position of one short circuit.

For example, at an operating frequency of 75 Gc, if one waveguide section is WR-15 and the other is WR-12, $\lambda_{G1}=0.5230$ cm, and $\lambda_{G_2}=0.4719$ cm. A displacement of 0.2615 cm will produce a phase shift of 39 degrees, which is approximately one ninth of the phase shift that would be produced by a single phase shifter using WR-12 waveguide.

Another application is in the investigation of uniformity of waveguide sections and the suitability of short circuits for phase shift standards. If the arrangement of Fig. 2 is used and the two waveguide sections are nominally identical, there will ideally be zero phase shift of the output as the short circuits are moved. Any phase shift which actually does occur is due to deficiencies in the short circuits or the waveguide sections, or in both.

Another application might consist of the determination of relative displacement from the measurement of phase shift. This would require that the motion of the two short circuits be independent rather than ganged. The sensitivity of phase shift to relative displacement could be preselected by choosing the cutoff frequencies of the individual waveguides as desired.

If the tuners are dispensed with, the differential phase shifter will still function but with reduced accuracy, due to the finite directivities of the directional couplers and the reflections in the system.

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An Extension of the TE_{01n} Resonator Method of Making Measurements on Solid Dielectrics

Dielectric measurements using cylindrical cavities resonating in the TE_{01n} mode are discussed by several authors.[1-4] The cavity is physically arranged as illustrated in Fig. 1, and the dielectric properties of the specimen are determined from three experimentally observed variables: the resonant frequency, the phase shift constant of the empty resonator and the Q of the cavity. The dielectric constant and loss tangent of the specimen are then calculated using formulas which are strictly valid only if the losses of the sample-filled cavity are very small.

The purpose of this communication is, first, to show that it is possible to remove the restriction of low-loss samples and, second, to suggest a departure in the experimental procedure which eliminates the necessity of performing the usual quality-factor measurements.

Fig. 1 shows a right-circular cylindrical cavity, of radius a and length l, partially filled with a disc-shaped dielectric slab of thickness d, permittivity ϵ and conductivity σ. The problem is to evaluate ϵ and σ from measured quantities that are to be specified later. As with the investigation of any resonant structure, the present study is concerned with three fundamental issues: the normal modes or free oscillations of the cavity, its forced oscillations and resonance. In what follows these issues are discussed in that order.

The normal modes of the sample-filled cavity are solutions of Maxwell's equations subject to conditions that must be satisfied at the boundaries. Thus, if the resonator has perfectly conducting walls, the general

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[1] A. Von Hippel, "Dielectric Materials and Applications," John Wiley and Sons, Inc., New York, N. Y., pp. 63–122; 1954.
[2] F. Horner *et al.*, "Resonance methods of dielectric measurement at centimeter wavelengths," part III, *J. Inst. Elec. Engrs. (London)*, vol. 93, pp. 53–68; January, 1946.
[3] C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., p. 625; 1947.
[4] L. Hartshorn and J. A. Saxton, "The dispersion and absorption of electromagnetic waves," in "Encyclopedia of Physics," Springer-Verlag, Berlin, Germany, pp. 679–685; 1958.

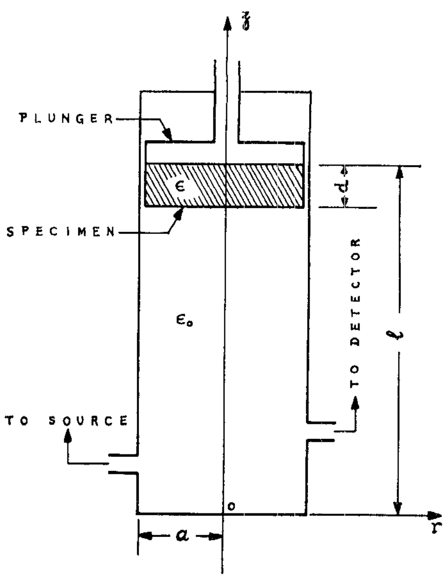

Fig. 1—TE_{01n} resonant cavity for measuring the dielectric properties of solids.

TE_{01} field configuration in the dielectric region may be specified by the familiar relations

$$\begin{aligned} E_\theta &= \frac{s\mu}{h} J_0'(hr)\left[Ce^{-\gamma z} + C'e^{\gamma z}\right]e^{st} \\ H_r &= -\frac{\gamma}{h} J_0'(hr)\left[Ce^{-\gamma z} - C'e^{\gamma z}\right]e^{st} \\ H_z &= J_0(hr)\left[Ce^{-\gamma z} + C'e^{\gamma z}\right]e^{st} \\ h^2 - \gamma^2 &= -s^2\mu\epsilon_c \\ h &= 3.832/a \\ \epsilon_c &= \epsilon + \sigma/s \end{aligned} \tag{1}$$

where the real and imaginary parts of the frequency $s=-\omega_i+j\omega$ and those of the propagation factor $\gamma=\alpha+j\beta$ are as yet unknown. In the sample-free portion of the cavity, the corresponding field relations are

$$\begin{aligned} E_{\theta_0} &= \frac{s\mu_0}{h} J_0'(hr)\left[C_0e^{-\gamma_0 z} + C_0'e^{\gamma_0 z}\right]e^{st} \\ H_{r_0} &= -\frac{\gamma_0}{h} J_0'(hr)\left[C_0e^{-\gamma_0 z} - C_0'e^{\gamma_0 z}\right]e^{st} \\ H_{z_0} &= J_0(hr)\left[C_0e^{-\gamma_0 z} + C_0'e^{\gamma_0 z}\right]e^{st} \\ h^2 - \gamma_0^2 &= -s^2\mu_0\epsilon_0 \\ h &= 3.832/a \\ \gamma_0 &= \alpha_0 + j\beta_0. \end{aligned} \tag{2}$$

In (2), ϵ_0 denotes the permittivity of the empty space and, except in the case of the Bessel function, the subscript zero refers to quantities pertaining to the empty portion of the sample-loaded cavity.

Boundary conditions require that the tangential component of $\boldsymbol{E}$ and the normal component of $\boldsymbol{B}$ must vanish at $z=0$ and at $z=l$. Also, the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ and the normal component of $\boldsymbol{B}$ must be continuous at $z=l-d$. Applying these conditions yields

$$\frac{\mu_0}{\gamma_0}\tanh\gamma_0(l-d) + \frac{\mu}{\gamma}\tanh\gamma d = 0. \tag{3}$$

Eq. (3) expresses mathematically the condition for free oscillations. Note that, when